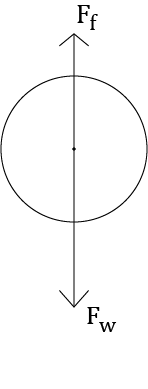
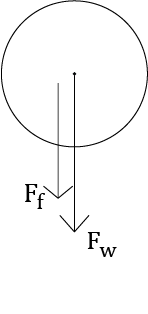
Quick Air Resistance Calculus

Remember this free body diagram from my air resistance paper?



Well, that was with the ball falling. What if I throw it up?



Now, the forces point the same way, so we’ll get a different answer. Let’s do the calculus. First, set up the equation and set up a differential equation to solve for . Also, we will define down as negative since that is how I did it in the original paper. Assume we are given and .

Now, unlike before, the right-hand integral is a more familiar integral than hyperbolic functions are. We can utilize the following integral rule:

The right-hand integral looks similar to that, except we have tricky constants in there to deal with. Notice that because of those constants, the general form of our integral doesn’t look like the rule. There is no constant in front of the . So, let’s factor out the and put it on the left side. Also note that the left side is solved because it is a simple integral. The +C will just be added to the right side though, so we’ll ignore it for now.

Now, let’s identify the parts of our integral. The is our , so . Our is just , so . Now, let’s plug that into our rule to get an answer.

Note that the equation was cleaned up a bit by simplifying fractions. We now have to solve for our constant, our +C. We are given a , so we know that .

Now, we have to solve this equation for . Look at the 4th equation in the work above where the value for C was moved to the left side. If we take of both sides, we’ll cancel out the and on the right side and we can easily solve for . However, the left side is more complicated. It can be dealt with though by using the following identity:

So, looking at the left side, we’ll get the following.

In the second step, the top and bottom were multiplied by to eliminate complex fractions. Now, let’s plug this back in and solve for .

That’s our velocity equation! I checked it out in desmos and integral-calculator, so for now while I haven’t written a program, that suffices for me to know that that’s correct. It’s always good to gut check by testing at extremes though. What happens when ? Well, that ’s go away and we/re left with , which cancels out to . So that’s good. Another good gut check would be to graph this equation and the velocity equation from last time for falling from rest, . When our equation equals 0, aka when the force of friction transitions from going downwards to going upwards and we change velocity equations, both equations seem to have the same slope, meaning they smoothly and continuously transition into each other.

Now, what about acceleration? Well, let’s take the derivative! It’s a really messy derivative though. Like, really messy. As a matter of fact, I just realized I did it wrong, so I have to fix it tomorrow.

What about position? Well, currently I’m stuck on what to do with the integral. I solved a general case of , but the constants in the equations make this tricky, so I’m not sure right now. I checked with an integral calculator and there is an answer, so it’s not like this integral has no answer, it can be done.

UPDATE

I figured out acceleration and position. For acceleration, let’s go through the messy derivative. We first do quotient rule.

Note what happened here. I wrote out the result from quotient rule in order. I’ll highlight it on color for you to see easier.

Now, let’s factor stuff out to simplify this equation. Both terms in the numerator have a , so let’s factor that out.

Now, let’s analyze the numerator. The two terms will cancel out with each other. So, we’re left with the following. Let’s simplify it and get a final equation.

So, this is our equation for acceleration. Again, as a good gut check, let’s test this at extremes. At , we know that , so using our original formula derived from , we can solve for

If we plug , into our formula, we get the following.

So, we know that that works. Another point where we could test it is when . At this point, there is no air resistance, so the acceleration should just be . This is more tedious to solve, so I will leave it as a challenge for you. However, I can confirm that you do get as your answer when you do the work because I did check it.

Now, what about the position equation? Well, let’s integrate the velocity equation.

This integral isn’t obvious at first. Let’s make this look neater by doing a u-sub to get rid of the .

Now, I solved this integral by converting everything to and . After making into , the next step is to multiply the whole fraction by in order to get rid of the complex fraction that resulted.

Now, with a bit of a clever u-sub, we can solve this. Notice that, if we ignore all the constants, the derivative of the denominator is the numerator. . However, the constants mess it up. Let’s try it anyway and see if we can do something.

For us to be able to immediately substitute this in, the numerator would have to match the result we just found. The coefficient in front of the terms must be the same, as with those in front of the terms. However, we can clearly see that they don’t. The numerator has coefficients of and whereas our substitution has coefficients of and . However, it could be possible that by taking out a constant from the numerator, we could make it into our substitution expression. Let’s find out and see if the coefficients have a common factor.

Our two sets of coefficients have the same common factor between themselves! That means if we factor out a from the numerator, we can do our u-sub (or technically w-sub).

Now look at that, our is now in the numerator! You can verify this for yourself by multiply the numerator by the in the second equation to ensure you get the first equation back as your answer. Now, let’s perform our w-sub and finally solve the integral.

Now, let’s substitute back in for and then for .

Lastly, we just have to solve for the . Let’s assume that at , the ball starts at . We can do the following.

If we start at , the becomes . However, for possible future problems, we can be given any to start at and it won’t always be . Note that we’re gonna simplify this equation a bit, so we’ll keep the radical inside the . Our final equation is:

Now, let’s put all of these equations together. However, I’m gonna mess with the velocity and acceleration equations a little bit. Using and seems to be much more natural than using , so let’s convert everything to that now that we’re aware of that.

So, here are our final equations.

I may make an analysis of these equations in another update.